Defining Brushless DC Motors Blog #3 of 4:
Deriving Kt motor sensitivity & Kb back emf constants

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In my first two blogs I presented an overview of a simple six-step procedure for defining a permanent magnet BDCM and showed how to establish the relationship between motor sensitivity (Kt) and back emf (Kb). In this third blog I will show how to define the relationships between the motor constants and then derive the actual motor constants.

Recalling EQU (9) from our last blog in this series

$$Kt = 1.352\ K_b \qquad \text{EQU (10)}$$

In a DC current motor,

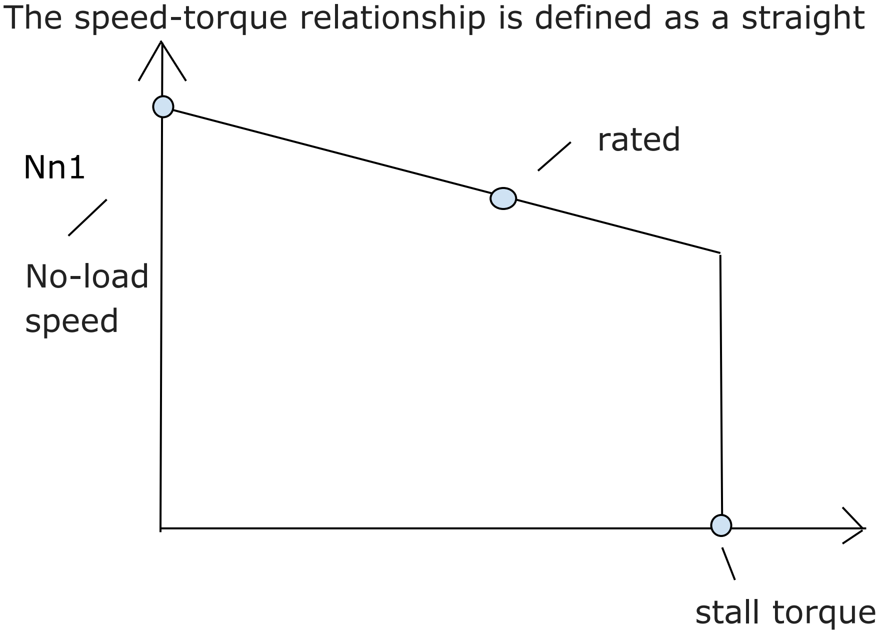
$$V_L = E + IR \qquad \text{EQU (11)}$$

which indicates that the excitation voltage (VL) is equal to the emf voltage produced (E) plus the voltage in the armature (IR).

or

$$E = V_L - IR \qquad \text{EQU (12)}$$

The speed-torque relationship is defined as a straight line:



The equation defining this straight line is:

$$N = N_{n1} - mT \qquad \text{EQU (13)}$$

where

N = motor speed

N_{n1} = the motor no load speed

$m = \frac{\Delta n}{\Delta T}$ = the algebraic slope of the line

T = torque

Substituting EQUS (12) and (13) into EQU (8) we get:

$$Kb = \frac{V_L - IR}{mT + N_n 1} \qquad \text{EQU (14)}$$

from EQU (7):

$$I = \frac{T}{Kt}$$

$$\therefore Kb = \frac{V_L - RT/K_t}{N_n 1 - mT} \qquad \text{EQU (15)}$$

rewriting:

$$V_L - \frac{R}{K_t}\mathrm{T}(N_n 1 - mT)^{-1} = K_b \qquad \text{EQU (16)}$$

In EQU (16) $VL, R,$ and N_{n1} are known constants. What we need to know is values of K_t and m (slope of speed line) such that K_b is constant for all values of T.

Recall EQU (6) $K = \frac{N_s P}{2\Pi}$, a constant,

and recall Φ, an air gap flux, is constant for a permanent magnet DC current motor. Therefore, K_b must be constant by definition. $(K_b = K\Phi)$.

Values of K_t and m such that K_b is constant can be determined by differentiating EQU (16) as follows:

$$\frac{d}{dT}\left(V_L - \frac{R}{K_t}\ T\right)(N_{n1} - mT)^{-1} = \frac{d}{dT}K_b = 0 \qquad \text{EQU (17)}$$

$$\frac{-R}{K_t}(N_{n1} - mT)^{-1} + \left(V_L - \frac{R_T}{K_t}\right)(-1)(N_{n1} - mT)^{-2}(-m) = 0 \qquad \text{EQU (18)}$$

Rewriting

$$\frac{-R/K_t}{N_{n1}-mT} + \frac{m\ (V_L - RT/K_t)}{(N_{n1}-mT)^{\ 2}} = 0 \qquad \text{EQU (19)}$$

multiply by $(N_{n1} - mT)$:

$$-R/K_t + m\frac{(V_L - RT/K_t)}{(N_{n1}-mT)} = 0 \qquad \text{EQU (20)}$$

$$\therefore \frac{R}{mK_t} = \frac{V_L - RT/K_t}{N_{n1}-mT}\ \mathrm{T} \qquad \text{EQU (21)}$$

substituting EQU (21) into EQU (15) we get:

$$K_t = \frac{R}{K_b m} \qquad \text{EQU (22)}$$

recall EQU (15):

$$K_b = \frac{V_L - RT/K_t}{N_{n1}-mT} \qquad \text{EQU (15)}$$

for no load, T = 0;

$$K_b = \frac{V_L}{N_{n1}} \qquad \text{EQU (23)}$$

Therefore, by knowing the torque rate requirement (m) and the excitation voltage (VL) and by knowing the motor envelope (which dictates the R value) one can size a motor by making use of EQUS (10), (22), and (23):

$$\frac{MOTOR}{SENSITIVITY} \quad K_t = \frac{R}{K_b m} \frac{in-oz}{Amp} \qquad \text{EQU (22)}$$

R= motor resistance

m= slope of speed-torque line = $\frac{\Delta n}{\Delta T} \frac{RPM}{in-oz}$

$$\frac{BACK\ EMF}{CONSTANT,} \quad K_b = \frac{V_L}{N_{n1}} \frac{volts}{1000\ rpm} \qquad \text{EQU (23)}$$

V_L = excitation voltage

N_{n1}= no load speed of motor

Electrical energy converted to mechanical energy.

$$K_t = 1.352\ K_b \qquad \text{EQU (10)}$$

More to come

After outlining a simple 6 step procedure we defined the relationships between the motor constants and then derived the actual motor constants. In our 4th and final blog in this series we give a specific example.

Stay tuned!

Let's Talk

I am confident that Composite Motors, Inc. with their Team and resources would be an essential foundation for supporting your BDCM development. All of Composite Motors, Inc.'s products are made on American soil. Let's talk! Email me at johnk@jdtechsales.com or call me at 781-864-2220 or visit Composite Motors, Inc.'s website at www.compositemotors.com

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